

# Letters

## Comments on "Attenuation Characteristics of Hollow Conducting Elliptical Waveguides"

G. FALCIASECCA, C. G. SOMEDA, AND F. VALDONI

In the above paper,<sup>1</sup> attenuation constants for several modes of a metal elliptical waveguide are computed by means of two basic formulas [footnote one, eqs. (1) and (4)]. These expressions do not coincide with those that Chu obtained long ago [1]. Several numerical discrepancies are pointed out.<sup>1</sup>

As Kretzschmar states, the standard first-order powerloss method for the attenuation in metal waveguides is very well known. As the partial steps are not reported in Kretzschmar's paper, it has to be inferred from the above-mentioned equations that the following quantity has been used as the real part of the wall impedance:

$$R = (\pi\mu f/\sigma)^{1/2} \quad (1)$$

$$\alpha_c \sqrt{a^3 \sigma} = \left( \frac{\pi \epsilon_0 \mu f_0}{1 - (f_c/f_0)^2} \right)^{1/2} C_{e_m}^2(\xi_0, q) \frac{\frac{4q}{e^2} \left( \frac{f_c}{f_0} \right)^2 (1 - e^2) \int_0^{2\pi} C_{e_m}^2(\eta, q) d\eta + [1 - (f_c/f_0)^2] \int_0^{2\pi} C_{e_m}^2(\eta, q) d\eta}{2\sqrt{1 - e^2} \int_0^{2\pi} \int_0^{\xi_0} [C_{e_m}^2(\xi, q) C_{e_m}^2(\eta, q) + C_{e_m}^2(\xi, q) C_{e_m}^2(\eta, q)] d\xi d\eta}.$$

where  $f$  is the frequency,  $\mu$  is the permittivity, and  $\sigma$  is the dc conductivity.

It is very well known that (1) holds in planar geometry and is also applicable to circular waveguides. It seems that the above-mentioned equations have been obtained by analogy with these cases.

A recent paper [2], which was published shortly after the date of Kretzschmar's original manuscript, follows a different path. The wave equation is solved for radial propagation in elliptical coordinates; then, as usual, the displacement current is neglected compared with the conduction current. Thus a longitudinal wall impedance

$$Z_s = (1 + j)Rb/h_t \quad (2)$$

and a transverse wall impedance

$$Z_\eta = (1 + j)Rb/h_t \quad (3)$$

are obtained, where  $2b$  is the minor axial length in the cross section of the elliptical waveguide and  $h_t$  is the first metric coefficient in the elliptical coordinate frame, evaluated on the metal wall. The details of the derivation are contained in [2].

From (2) and (3) one gets back to (1) if and only if the ellipse is indeed a circle, because then  $h_t = b$ . On the other hand, when the complete expressions (2) and (3) are introduced in the standard power-loss method, or in another first-order perturbation approach [3], then Chu's formulas [1] are obtained. Note that their original derivation had been performed by matching the fields on the elliptical boundaries.

Despite the little amount of experimental work that we are aware of, we trust Chu's formulas as being better grounded than those used by Kretzschmar. Therefore, we suggest that the very appreciable numerical evaluations of normalized attenuation charts, done in Kretzschmar's paper, be extended to those formulas and practical consequences of the different approach be pointed out.

### REFERENCES

- [1] L. J. Chu, "Electromagnetic waves in elliptic hollow pipes of metal," *J. Appl. Phys.*, vol. 9, pp. 583-591, Sept. 1938.
- [2] G. Falciasecca, C. G. Someda, and F. Valdoni, "Wall impedances and application to long-distance waveguides," *Alta Frequenza*, vol. 40, pp. 426-434, May 1971.
- [3] R. E. Colin, *Field Theory of Guided Waves*. New York: McGraw-Hill, 1960, sect. 5.3.

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<sup>1</sup> J. G. Kretzschmar, *IEEE Trans. Microwave Theory Tech.*, vol. MTT-20, pp. 280-284, Apr. 1972.

### Reply<sup>2</sup> by Jan G. Kretzschmar<sup>3</sup>

The only, but important, difference between Chu's formulas [1] and the ones given in [2] is obvious when the former are rewritten under the following normalized form.

For even TM modes:

$$\alpha_c \sqrt{a^3 \sigma} = \left( \frac{\pi \epsilon_0 \mu f_0}{1 - (f_c/f_0)^2} \right)^{1/2} \frac{C_{e_m}^2(\xi_0, q) \int_0^{2\pi} C_{e_m}^2(\eta, q) d\eta}{2\sqrt{1 - e^2} \int_0^{2\pi} \int_0^{\xi_0} [C_{e_m}^2(\xi, q) C_{e_m}^2(\eta, q) + C_{e_m}^2(\xi, q) C_{e_m}^2(\eta, q)] d\xi d\eta}.$$

For even TE modes:

$$\alpha_c \sqrt{a^3 \sigma} = \left( \frac{\pi \epsilon_0 \mu f_0}{1 - (f_c/f_0)^2} \right)^{1/2} \frac{\frac{4q}{e^2} \left( \frac{f_c}{f_0} \right)^2 (1 - e^2) \int_0^{2\pi} C_{e_m}^2(\eta, q) d\eta + [1 - (f_c/f_0)^2] \int_0^{2\pi} C_{e_m}^2(\eta, q) d\eta}{2\sqrt{1 - e^2} \int_0^{2\pi} \int_0^{\xi_0} [C_{e_m}^2(\xi, q) C_{e_m}^2(\eta, q) + C_{e_m}^2(\xi, q) C_{e_m}^2(\eta, q)] d\xi d\eta}.$$

It is now clear that the factor  $\text{th } \xi_0 = \sqrt{1 - e^2}$  in these equations replaces the factor  $\sqrt{1 - e^2} \cos^2 \eta$  in the corresponding formulas in [2]. This is due to the fact that the wall impedance has been taken equal to  $(\pi\mu f/\sigma)^{1/2}$ , as was pointed out by Falciasecca *et al.* A comparative study of both sets of formulas is indeed very interesting, and I hope to present the first results in the near future. Meanwhile, I would like to point out that it is not shown in [3] how the fields inside the elliptical waveguide and the fields in the metal wall can be matched at the boundary  $\xi = \xi_0$ . Another interesting problem is the accuracy of the asymptotic formulas for the modified Mathieu of the fourth kind and the approximations for their first derivative.

### REFERENCES

- [1] L. J. Chu, "Electromagnetic waves in elliptic hollow pipes of metal," *J. Appl. Phys.*, vol. 9, pp. 583-591, Sept. 1938.
- [2] J. G. Kretzschmar, "Attenuation characteristics of hollow conducting elliptical waveguides," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-20, pp. 280-284, Apr. 1972.
- [3] G. Falciasecca, C. G. Someda, and F. Valdoni, "Wall impedances and application to long-distance waveguides," *Alta Frequenza*, vol. 40, pp. 426-434, May 1971.

<sup>2</sup> Manuscript received August 7, 1972.

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## Focusing of 104-GHz Beams by Cylindrical Mirrors

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The use of cylindrical mirrors to focus 52-GHz beams over a 85-m-long path has been previously reported [1]. We shall report in this letter the results of tests made at 52 and 104 GHz over an extended 350-m-long path incorporating 10 refocusers (20 mirrors). The arrangement is shown in Fig. 1.

The round-trip loss measured in clear weather is 2.3 dB at 52 GHz and 1.0 dB at 104 GHz. These losses are exclusive of the launching and collecting-dish losses, the Mylar-window losses, and the absorption by the oxygen line of the atmosphere. The lower loss observed at 104 GHz can be accounted for by a reduced spillover at the

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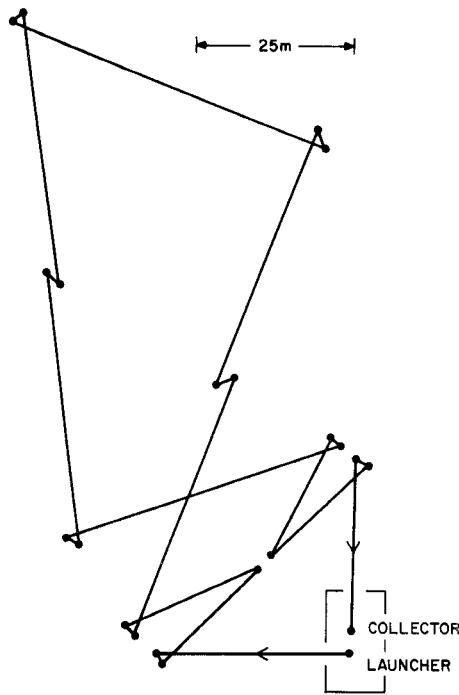


Fig. 1. Outline of the experimental path setup at Crawford Hill, Holmdel, N.J. This path incorporates 10 focusers. The dots along the broken line indicate mirror locations.

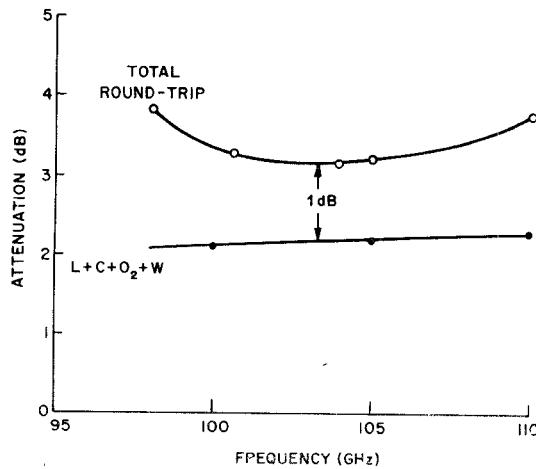


Fig. 2. The top curve gives the measured total round-trip attenuation in decibels as a function of frequency. The lower curve is the sum of the attenuation measured when the launcher and collector face each other (1.8 dB at 104 GHz), the Mylar window loss (0.2 dB), and the attenuation by the atmospheric oxygen (0.23 dB at 104 GHz).

mirror edges and the fact that smaller portions of the cylindrical mirrors are used at the higher frequency (degradations resulting from the lack of uniformity in mirror curvature are lessened if the beam size is reduced). When going from 52 to 104 GHz the only changes that need to be made are substitution of new feeds and optimization of the curvature of the cylindrical mirrors. The same launching and collecting dishes, made of spun aluminum, are used at both frequencies.

The variation of the measured round-trip loss as a function of frequency is shown in Fig. 2. This figure shows that the loss does not exceed 1.6 dB over a 10-percent bandwidth centered at 104 GHz. This loss is significantly higher than the theoretical ohmic loss ( $\sim 0.1$  dB). Yet it is negligibly small compared with losses resulting from heavy rains. To determine the losses due to the mirror surfaces being wet, a mirror was thoroughly sprayed with water. The attenuation increases by 0.1 dB. Recovery occurs 30 s after the termination of the spraying under average wind conditions.

The variations in transmission due to wind are of the order of 0.2 dB for 8–16-km/h winds and 0.5 dB for 24–32-km/h winds. Gusts of 55-km/h winds can produce 2-dB variations.

Slow daily variations, not exceeding 3 dB, are observed at 52 and 104 GHz; they are attributed to thermal changes resulting in deformations of the supporting structures (made of steel pipes).

In conclusion, we have shown that commercial (1-m by 1-m) glass mirrors can be used for guiding and directing 104-GHz beams with little losses.

#### ACKNOWLEDGMENT

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#### REFERENCES

[1] J. A. Arnaud, D. C. Hogg, and J. T. Ruscio, "Focusing of 52-GHz beams by cylindrical mirrors," *IEEE Trans. Microwave Theory Tech. (Short Paper)*, vol. MTT-20, pp. 344–345, May 1972.

### Fast Parameters Calculation of the Dielectric-Supported Air-Strip Transmission Line

EUGENIO COSTAMAGNA

The method presented by Smith in [1] for the evaluation of the fringing capacitances in microstrip and suspended substrate structures is very useful in analysis and optimization problems and permits fast and accurate calculations.

We have applied the method to the structures, with electric or magnetic side walls, shown in Fig. 1, which differ from those considered in [1] because at the bottom a magnetic wall instead of an electric one is assumed.

The fast calculation of these structures is of great interest and includes as a particular case ( $g_1=0$ ) the analysis of the dielectric-supported air-strip transmission line in Fig. 2.

In his paper [1] Smith expresses the capacitance of the line as a slowly converging series. Convergence is obtained by subtracting from it, term by term, a second series representing the capacitance (Smith's  $C_T$ ) in a convenient structure, in which the charge distribution (Smith's  $\rho(x)$ ) is similar and can be readily found together with the capacitance from conformal mappings.

In our case, a convenient series expression for the capacitance and a trial function for the charge distribution are obtained from the structure with homogeneous dielectric in Fig. 3. The capacitance and the charge distribution for this geometry can be found from Smith's conformal mappings applied to the new geometry in Fig. 4.

This is because Green's functions for the geometry in Fig. 3 and the geometry in Fig. 4 (which differs from that introduced by Smith for the geometrical dimensions only), calculated on the center conductor, are identical, as will be shown in the following paragraph.

In the case of electric side walls, with homogeneous dielectric, it is sufficient to compare Green's functions for a half section in Fig. 3 [2] and for a half section in Fig. 4 [3]. For the first half section, with the notation introduced in [2], we have:

$$G_0 = \frac{2}{\pi \epsilon_0} \sum_{1,3,5,\dots}^{\infty} \frac{\sinh \frac{m\pi}{2a} g_2 \cosh \frac{m\pi}{2a} g_2 \sin \frac{m\pi}{2a} x \sin \frac{m\pi}{2a} \xi}{\frac{m}{2} \cosh \frac{m\pi}{2a} 2g_2}$$

and for the second half section:

$$G_0 = \frac{2}{\pi \epsilon_0} \sum_{1,3,5,\dots}^{\infty} \frac{\sinh \frac{m\pi}{2a} 2g_2 \sinh \frac{m\pi}{2a} 2g_2 \sin \frac{m\pi}{2a} x \sin \frac{m\pi}{2a} \xi}{\frac{m}{2} \sinh \frac{m\pi}{2a} 4g_2}$$

The two expressions, as seen by inspection, coincide, and a variational